



**University
of Victoria**

Graduate Studies

Notice of the Final Oral Examination
for the Degree of Doctor of Philosophy

of

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MSc (Simon Fraser University, 2012)

BSc (University of Victoria, 2009)

“Irreversible k - Threshold Conversion Processes on Graphs”

Department of Mathematics and Statistics

Tuesday, April 3, 2018

1:30 P.M.

Clearihue Building

Room B017

Supervisory Committee:

Dr. Kieka Mynhardt, Department of Mathematics and Statistics, University of Victoria (Supervisor)

Dr. Peter Dukes, Department of Mathematics and Statistics, UVic (Member)

Dr. Venkatesh Srinivasan, Department of Computer Science, UVic (Outside Member)

External Examiner:

Dr. Nancy Clarke, Mathematics and Statistics, Acadia University

Chair of Oral Examination:

Dr. Simon Devereaux, Department of History, UVic

Dr. David Capson, Dean, Faculty of Graduate Studies

Abstract

Given a graph G and an initial colouring of its vertices with two colours, say black and white, an irreversible k -threshold conversion process on G is an iterative process in which a white vertex becomes permanently coloured black if at least k of its neighbours are coloured black at time $t - 1$. A set S of vertices is an irreversible k -threshold conversion set (k -conversion set) of G if the initial colouring in which the vertices of S are black and the others are white results in the whole vertex set becoming black eventually. In the case where G is $(k + 1)$ -regular, it can be shown that the k -conversion sets coincide with the so-called feedback vertex sets, or decycling sets.

In this dissertation we study the size, $ck(G)$, and structure of minimum k -conversion sets in several classes of graphs, G . We examine conditions that lead to equality and inequality in existing bounds on $ck(G)$ for k - and $(k+1)$ -regular graphs. Furthermore, we derive new sharp lower bounds on $ck(G)$ for regular graphs of degree ranging from $k+1$ to $2k-1$ and for graphs of maximum degree $k+1$, and we determine exact values of $ck(G)$ for certain classes of trees.

We show that every $(k + 1)$ -regular graph has a minimum k -conversion set that avoids certain structures in its induced subgraph. These results lead to new proofs of several known results on colourings and forest partitions of $(k + 1)$ -regular graphs and graphs of maximum degree $k + 1$.